AN INVARIANT OF CAUCHY-RIEMANN SEIFERT 3-MANIFOLDS AND APPLICATIONS

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ABSTRACT. We compute a recently introduced geometric invariant of stricly pseudoconvex CR 3-manifolds for certain circle invariant spherical CR structures on Seifert manifolds. We give applications to the problem of filling the CR manifold by a complex hyperbolic manifold, and more generally by a Kähler-Einstein or an Einstein metric.

1. INTRODUCTION.

In [7] we introduced a new invariant, called the ν-invariant, of strictly pseudoconvex Cauchy-Riemann (CR) compact 3-manifolds. This invariant is an analogue in CR geometry of the η-invariant in conformal geometry. The definition of the ν-invariant is rather abstract and makes it difficult to get explicit expressions. The aim of this paper is to provide a computation of ν in the simple, yet interesting, case of certain spherical $\mathbb{S}^1$-invariant CR structures on Seifert manifolds, and to deduce some geometric applications.

The spherical (that is, locally isomorphic to the standard CR 3-sphere) CR structures we are interested in come with an action of $\mathbb{S}^1$ without fixed point and transverse to the contact distribution (see [12] for a classification of all spherical $\mathbb{S}^1$-invariant CR structures). They appear as orbifold $\mathbb{S}^1$-bundles over 2-dimensional orbifolds. At each orbifold point, the orbifold data consists of the following: the local fundamental group is $\mathbb{Z}/\alpha\mathbb{Z}$ ($\alpha \in \mathbb{N}^*$), and a generator acts on a local chart around p of the basis manifold as $e^{i\frac{2\pi\beta}{\alpha}}$ and on the fiber as $e^{i\frac{2\pi\gamma}{\alpha}}$ with β and γ prime to α. The orbifold $\mathbb{S}^1$ bundle is topologically classified by the degree (first Chern number) d of the bundle—in this case a rational number. We then endow the manifold with an invariant strictly pseudoconvex CR structure: The underlying contact structure is provided by a constant curvature equivariant connection 1-form on the bundle, whereas the complex structure is induced from the basis

Riemann surface. The pseudoconvexity condition constrains the degree d to be negative. Our main result then reads:

1.1. Theorem. *Let X be a compact spherical Cauchy-Riemann 3-manifold which is a $\mathbb{S}^1$ orbifold bundle of degree $d < 0$ over a compact orbifold Riemann surface Σ of Euler characteristic χ (a rational number). The ν-invariant of X is*

$$(1) \qquad \nu(X) = -d - 3 - \frac{\chi^2}{4d} - 12 \sum_{j=1}^{p} s(\alpha_j, \beta_j, \gamma_j)$$

where $s(\alpha, \beta, \gamma)$ is the Dedekind sum $\frac{1}{4\alpha} \sum_{k=1}^{\alpha-1} \cot\left(\frac{k\beta\pi}{\alpha}\right) \cot\left(\frac{k\gamma\pi}{\alpha}\right)$.

We first remark that for this class of CR manifolds, the ν-invariant depends only on the topology, and not, for instance, on the basis complex structure. This is *a priori* known, since the gradient of ν is the Cartan curvature [7, Theorem 8.1], which vanishes for spherical CR manifolds.

We now pass to geometric applications. Such X being locally isomorphic to the standard CR sphere $\mathbb{S}^3$, it is the boundary at infinity of a complex hyperbolic metric defined in a neighbourhood $(0, \varepsilon] \times X$ of X (in the case of the 3-sphere we get the Bergmann metric on the 4-ball).

From [7, Theorem 1.2], we get the following obstruction for this neighbourhood to have a global extension to a smooth complex hyperbolic surface (with only one end):

1.2. Corollary. *Let X be as in Theorem 1.1. If X^3 is the boundary at infinity of a complex hyperbolic metric defined on the interior M of a smooth compact manifold $\bar{M}^4$ with boundary X, then one has necessarily $\nu(X) = -\chi(\bar{M}) + 3\tau(\bar{M})$, where $\chi(\bar{M})$ and $\tau(\bar{M})$ denote the Euler characteristic and signature of $\bar{M}$. In particular, $\nu(X)$, as given by the formula (1), is an integer.*

We can restate this in a special case :

1.3. Corollary. *Let X be a $\mathbb{S}^1$-bundle of degree d over a Riemann surface Σ of Euler characteristic χ, with a $\mathbb{S}^1$-invariant spherical CR structure. If $\frac{\chi^2}{4d}$ is not an integer then X is not the boundary at infinity of a complex hyperbolic metric.*

The case $d = \frac{\chi}{2}$ yields an integer, and indeed, if Σ is hyperbolic, $\bar{M}$ can be taken to be the disk bundle of a square root of the tangent bundle of Σ, which is well known to carry a complex hyperbolic metric

issued from a representation of $\pi_1(\Sigma)$ in $SU(1,1) \subset SU(1,2)$. Our obstruction then gives an interesting hint on whether a CR flat Seifert 3-manifold may appear as a quotient of the complement of the limit set in the 3-sphere of some discrete fixed point-free subgroup of $SU(1,2)$ [1].

More generally, the calculation in Theorem 1.1 gives an obstruction for X to be the boundary at infinity of a Kähler-Einstein metric. The manifolds considered in this paper are known to bound a complex Stein space with at most a finite number of singular points [11] and one may wish to endow it with a Kähler-Einstein metric as in Cheng-Yau [10]. The type of metric to be considered has the same kind of asymptotic expansion near the boundary X as the Bergmann metric [3]; we called them "asymptotically complex hyperbolic" (ACH) in [7]. If no singular points are present and if the Cheng-Yau metric exists, one gets from the Miyaoka-Yau inequality the following:

1.4. Corollary. *Let X be as in Theorem 1.1. If X is the boundary at infinity of an ACH Kähler-Einstein metric on M^4, then*

$$\chi(\bar{M}) - 3\tau(\bar{M}) \geqslant -\nu(X) = d + 3 + \frac{\chi^2}{4d} + 12 \sum_{j=1}^{p} s(\alpha_j, \beta_j, \gamma_j).$$

This is a topological constraint on a filling. For more information on Stein fillings, see [16, 20]. We mention that this corollary remains true more generally for ACH Einstein (not Kähler) fillings, under the additional assumption that a Kronheimer-Mrowka contact invariant [14] of $(\bar{M}, X)$ is nonzero. This condition is fulfilled if M admits a symplectic form compatible with the contact structure on its boundary.

From [6, Theorem 5.12], one knows that pseudoconvex complex hyperbolic surfaces $\bar{M}$ have vanishing third homology group $H_3(\bar{M}, \mathbb{Z})$. Hence no multiple ends can occur, but one expects orbifold singularities or cusps to appear in the interior of a complex hyperbolic filling. The complex hyperbolic cusps can be compactified to yield a complex orbifold surface that we note again $\bar{M}$, by adding at the infinity of each cusp a quotient Σ_i of a 2-torus. The corollaries 1.2 and 1.4 remain true in this case, with the Euler characteristic and the signature of $\bar{M}$ replaced by their orbifold versions: In case ℓ cusps are present, there is an additional contribution in the signature coming from the self-intersection of each 2-torus at infinity. Namely, one has to consider the modified signature [4, proposition 3.4]

$$\tau_{cusp}(\bar{M}) = \tau(\bar{M}) - \frac{1}{3} \sum_{1}^{\ell} [\Sigma_i] \cdot [\Sigma_i].$$

Of course, corollary 1.3 is no more true, since the characteristic numbers are now rational; the denominator of ν only gives an hint on the order of the singularities needed to fill X.

Another important point is to compare these results with those obtained by use of the Burns-Epstein μ-invariant [8, 9] (it is already suggested at the end of [9] that obstructions follow from computations of μ). The μ invariant is defined on strictly pseudoconvex CR 3-manifolds with trivial tangent holomorphic bundle only. Roughly speaking, it comes from Chern-Simons-type constructions (integration of a local formula), whereas the ν-invariant is extracted from the Atiyah-Patodi-Singer η-invariant. The relation between μ and ν is similar to that between the η and the Chern-Simons invariants: More precisely [7, Theorem 1.3], when μ is defined, then for a CR structure J one has

$$\nu(J) \; = \; 3\,\mu(J) \; + \; \text{constant},$$

with the constant depending only of the underlying contact structure. Burns-Epstein's version of Miyaoka-Yau [9] then reads, if X is the boundary at infinity of a Kähler-Einstein M:

$$(2) \qquad\qquad \chi(\bar{M}) - \frac{1}{3}\,\bar{c}_1(\bar{M})^2 \geqslant -\mu(X),$$

with equality if the metric is complex hyperbolic; here $\bar{c}_1$ is a lift in $H^2(M, X)$ of $c_1(M)$.

A first important difference here is that our obstruction in Corollary 1.4 (filling by an ACH Kähler-Einstein metric) is purely topological, whereas (2) involves the complex structure.

Another important fact to be noticed, at least in the case when there is no orbifold quotient, is that the obstructions obtained by both methods are different: if X is a $\mathbb{S}^1$-bundle over the Riemann surface Σ, then the μ-invariant, being defined by a local formula, is multiplicative on finite coverings [8, 9]. Hence the values are

$$(3) \qquad\qquad \mu = \frac{\chi^2}{4d} \;\; \text{whereas} \;\; \nu = -\frac{\chi^2}{4d} - d - 3.$$

Equation (2) implies that 3μ must be an integer, *i.e.* $\frac{3\chi^2}{4d}$ must belong to $\mathbb{Z}$, a condition that is weaker than corollary 1.3, by a factor 3.

The reader will find the computation of ν in the next section. The beginning of section 3 is then devoted to a full explicitation of these results in an interesting particular case: that of lens spaces; formulas are given in Proposition 3.1. The paper then ends with the proof of the corollaries.

2. Computation of the invariant

We first describe our 3-dimensional compact strictly pseudoconvex CR manifold X in greater detail. The basis compact Riemannian orbifold Σ is endowed with a constant curvature metric and a complex structure. For any $\mathbb{S}^1$-bundle X over it of degree $d < 0$, there exists an equivariant connection 1-form $\bar{\theta}$ of constant curvature (relative to the basis metric). We shall take

$$\theta = \frac{1}{2}\bar{\theta}$$

as the contact form on X (the choice of the factor $\frac{1}{2}$, although slightly awkward at some places below, is made to ensure full consistency with the conventions of our previous work [7]); in this convention, fibers then have length π (rather than the more natural 2π). The complex structure J on the basis can be lifted on the contact ditribution H, and the bilinear form $\gamma = d\theta(J\cdot,\cdot)$ is an invariant metric on H under the circle action, which projects downwards as a constant multiple of the basis metric. From our normalization of contact form above, we have

$$\int_\Sigma d\theta = -\pi d, \quad \int_X \theta \wedge d\theta = -\pi^2 d.$$

The volume of the basis Σ endowed with the projected metric γ is $V = -\pi d$ and its curvature is $R = -\frac{2\chi}{d}$.

One then defines a sequence of Riemannian metrics

$$g_\rho = 4\rho^2\,\theta^2 + \gamma$$

on X, and one chooses an asymptotically complex hyperbolic Kähler-Einstein metric $\overline{g}$ (with scalar curvature -6) on $M = [r_0, +\infty[\times X$, defined as follows: the complex structure is trivially extended as

$$J_{|H} = J \ \text{ on } H, \quad J\partial_r = \mathrm{e}^{-r}\,\xi,$$

if ξ is the (vertical) Reeb field of θ. The Kähler form is (see [7]):

$$\bar{\omega} = \mathrm{e}^r\,(dr \wedge \eta + d\eta) - \frac{R}{2}\,d\eta - \frac{R^2}{12}\,\mathrm{e}^{-r}\,(dr \wedge \eta - d\eta) + o(\mathrm{e}^{-2r}).$$

It is explained in [7] why lower order terms in $\bar{\omega}$ are irrelevant in all what concerns the ν-invariant to be defined below. Let us denote its $(3,1)$ curvature tensor by R, the associated operator on 2-forms by R^{op} and denote by $\bar{\mathrm{n}}$ and $\mathbb{I}_r$ the outer unit normal and second fundamental form of the slice $\{r\} \times X$. We also need to define for a tensor F in $\otimes^3 T^*M$,

$$\mathfrak{S}(F)(X,Y,Z) = F(X,Y,Z) + F(Y,Z,X) + F(Z,X,Y),$$

and, if ρ is a 3-form and σ_1, σ_2, σ_3 are three vectors,

$$\mathscr{T}\left(\rho \otimes (\sigma_1 \otimes \sigma_2 \otimes \sigma_3)\right) = d\mathrm{vol}_{\{r\}\times X}\left(\sigma_1, \sigma_2, \sigma_3\right) \rho.$$

This definition immediately extends to a 3-form with values in $\otimes^3 TM$.

According to [7], the ν-invariant is obtained by taking the limit as r goes to infinity of the boundary contribution at $\{r\} \times X$ of the Atiyah-Patodi-Singer expression for the characteristic number $\chi - 3\tau$ of $[r_0, r] \times X$ with respect to the metric $\overline{g}$. This is explicited in the

2.1. **Definition.** The ν-invariant of X is

$$\nu(X) = \lim_{r \to +\infty} \nu(\overline{g}, r) = \lim_{r \to +\infty} \left\{ B(\overline{g}, r) - 3\,\eta(4\rho(r)^2\theta^2 + \gamma) \right\},$$

where $\rho(r) = \frac{1}{4} \mathrm{e}^r (1 + \frac{R}{2}\mathrm{e}^{-r} + \frac{R^2}{12}\mathrm{e}^{-2r})$, and $B(\overline{g}, r)$ is the integral term

$$B(\overline{g}, r) = -\frac{1}{12\pi^2} \int_{\{r\}\times X} \mathscr{T}(\mathbb{I}_r \wedge \mathbb{I}_r \wedge \mathbb{I}_r) + 3\,\mathscr{T}(\mathbb{I}_r \wedge R^{op}) - 3\,\mathfrak{S}(\mathbb{I}_r(., R(.,.)\bar{\mathrm{n}}).$$

It is shown in [7] that the limit converges and gives indeed rise to a CR invariant of X.

The η-invariant of (orbifold) circle bundles over (orbifold) Riemannian surfaces has been computed by Komuro [13] and more generally by Ouyang [18]. In our conventions, their results read:

2.2. **Theorem** (Ouyang). *The η-invariant of the metric $4\rho^2\theta^2 + \gamma$ on X is equal to*

$$\frac{1}{3}\left(d + 3 + 2d\left(\frac{\pi\rho^2}{V}\chi - \frac{\pi^2\rho^4}{V^2}d^2\right)\right) + 4\sum_{j=1}^{p} s(\alpha_j, \beta_j, \gamma_j).$$

In our setting, this yields:

$$3\,\eta(r) = -\frac{d}{8}\mathrm{e}^{2r} - \frac{\chi}{4}\mathrm{e}^r + \frac{7}{24}\frac{\chi^2}{d} + d + 3 + 12\sum_{j=1}^{p} s(\alpha_j, \beta_j, \gamma_j).$$

The local integral terms can now be computed by using the same techniques as in [7] and in [17]. These are lengthy but otherwise straightforward computations, and we give a few intermediate steps below.

The second fundamental forms are, up to order 2 terms,

$$\mathbb{I}_r = \left(1 + \frac{R^2}{8}\mathrm{e}^{-2r}\right)\mathrm{Id}_\xi + \frac{1}{2}\left(1 + \frac{R}{2}\mathrm{e}^{-r} + \frac{R^2}{8}\mathrm{e}^{-2r}\right)\mathrm{Id}_H + o(\mathrm{e}^{-2r})$$

(with ξ the Reeb form of the contact distribution) and the terms involving only $\mathbb{I}_r$ in the definition of $B(\overline{g}, r)$ are easily obtained from it;

one gets the contribution:

$$\left(\frac{3}{2}\,\mathrm{e}^{2r}+\frac{3}{4}R\,\mathrm{e}^{r}+\frac{R^2}{4}\right)\theta\wedge d\theta.$$

The curvature of the Kähler metric is formed from a constant term whose coefficient are exactly those of complex hyperbolic curvature in a $\overline{g}$-orthonormal basis on $[r_0,+\infty[\times X$ such as

$$\left(\partial_r,\mathrm{e}^{-r}(1-\frac{R^2}{12}\,\mathrm{e}^{-2r})^{-\frac{1}{2}}\xi,\right.$$
$$\left.\mathrm{e}^{-r/2}(1-\frac{R}{2}\,\mathrm{e}^{-r}+\frac{R^2}{12}\,\mathrm{e}^{-2r})^{-\frac{1}{2}}h,\mathrm{e}^{-r/2}(1-\frac{R}{2}\,\mathrm{e}^{-r}+\frac{R^2}{12}\,\mathrm{e}^{-2r})^{-\frac{1}{2}}Jh\right)$$

and an order 2 correction term whose effect on the sought formula is zero, see [7, proof of lemma 7.6] for a precise justification of this point. The contribution of the term involving R^{op} is then

$$3\left(-\frac{5}{4}\,\mathrm{e}^{2r}+\frac{R}{2}\,\mathrm{e}^{r}-\frac{7R^2}{48}\right)\theta\wedge d\theta,$$

and that of the last term is

$$-3\left(-\frac{1}{4}\,\mathrm{e}^{2r}+\frac{R}{4}\,\mathrm{e}^{r}-\frac{5R^2}{48}\right)\theta\wedge d\theta.$$

The final boundary term is then

$$B(\overline{g},r)=-\frac{d}{8}\,\mathrm{e}^{2r}-\frac{\chi}{4}\,\mathrm{e}^{r}+\frac{\chi^2}{24d}\ ,$$

whose divergent terms cancel exactly those of $3\eta(4\rho^2\theta^2+\gamma)$, as expected. Adding the constant terms yield Theorem 1.1, and the elementary explicit computations that follow it.

3. Explicitation for lens spaces and proof of the corollaries

We now specialize the formula obtained in Theorem 1.1 to lens spaces. We will then prove the corollaries in a second step.

The lens space $L(p,q)$ is the quotient of the 3-sphere $\mathbb{S}^3$ in $\mathbb{C}^2$ by $\mathbb{Z}/p\mathbb{Z}$, with its generator acting on $\mathbb{C}^2$ by $(\mathrm{e}^{\frac{2i\pi}{p}},\mathrm{e}^{\frac{2iq\pi}{p}})$, where q is prime with p. They are interesting in connection with filling by Einstein metrics, since some of them appear as boundary at infinity of seldual Einstein metrics [5]. On the other hand, it has been shown that large families of them admit symplectic fillings. In this case our result yields:

3.1. **Proposition.** *The ν-invariant of the lens space $L(p,q)$ is*

$$\nu(L(p,q))=-\frac{1}{p}+12\,s(p,q,1).$$

For sake of comparison, we recall to the interested reader the value of the classical η-invariant on lens spaces with the standard round metric, as computed by Atiyah-Patodi-Singer [2, Proposition 2.12]:

$$(4) \qquad\qquad \eta(L(p,q)) = -4\, s(p,q,1).$$

Proof. – For simplicity, we shall assume that $(q-1)$ is prime with p (as a matter of fact this implies that we take $q \neq 1$), and we leave the general case to the reader. Let us see the 3-sphere as the bundle $\mathscr{O}(-1)$ over the projective line $\mathbb{C}P^1$. The induced action on $\mathbb{C}P^1$ has two fixed points: the two antipodal points, with action of $\mathbb{Z}/p\mathbb{Z}$ generated by $\mathrm{e}^{\pm i2\pi \frac{q-1}{p}}$, and action in the fiber by $\mathrm{e}^{i\frac{2\pi}{p}}$ and $\mathrm{e}^{i2\pi \frac{q}{p}}$ respectively. Therefore $L(p,q)$ is a $\mathbb{S}^1$-orbifold bundle over an orbifold projective line with two orbifold points with angle $\frac{2\pi}{p}$. The Euler characteristic is $\chi = \frac{2}{p}$ and the Chern number is $d = -\frac{1}{p}$. Now our Theorem 1.1 and Ouyang's Theorem 2.2 give the formulas

$$\nu(L(p,q)) = -3 + \frac{2}{p} - 12\big(s(p,q-1,1) + s(p,1-q,q)\big),$$

$$\eta(L(p,q)) = 1 - \frac{1}{p} + 4\big(s(p,q-1,1) + s(p,1-q,q)\big).$$

We deduce $\nu(L(p,q)) = -\frac{1}{p} - 3\eta(L(p,q))$. The proposition then follows from (4). $\qquad\qquad\square$

For instance, there are large families of lens spaces that admit symplecting fillings [15], to which Corollary 1.4 may be applied.

Proof of the corollaries. Corollaries 1.2 and 1.4 rely on the formula discovered by the authors [7, Theorem 1.2]: for any Einstein asymptotically hyperbolic manifold (M^4, g),

$$(5) \quad \frac{1}{8\pi^2} \int_M \left(3|W^-|^2 - |W^+|^2 + \frac{1}{24}\,\mathrm{Scal}^2\right) - \chi(\bar M) + 3\,\tau(\bar M) = \nu(X).$$

For complex hyperbolic surfaces, the integral term is zero. If $\bar M$ is smooth, with X as the only end, then the topological contributions always are integers. Corollary 1.2 is then proved.

It is instructive to check the results for a holomorphic disk bundle D over a hyperbolic Riemann surface Σ, with X as its boundary. Clearly one has $\chi(D) = \chi(\Sigma) = \chi$ and $\tau(D) = -1$. If D carries a complex hyperbolic metric with X as its boundary at infinity, then corollary 1.2 gives the equation

$$\chi + 3 = -\nu(X) = d + 3 + \frac{\chi^2}{4d}$$

and the only solution is $d = \frac{\chi}{2}$. We then recover the well-known fact that the only disk bundles carrying a complex hyperbolic metric are the square roots of the (complex) tangent bundle.

Corollary 1.4 is again a direct consequence of (5), since for a Kähler-Einstein metric, the integral term is non negative. For an Einstein metric, the story is more complicated, but positivity is achieved if solutions to the Seiberg-Witten equations exist, and it is proven in [19, corollary 31] that it is a consequence of the nonvanishing of the Kronheimer-Mrowka invariants.

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